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Input-State Feedback Control of Switched Nonlinear Systems Using Multiple Lyapunov Functions

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Abstract. This paper is inclined to consider the problem of controller synthesis and to meet the goal of stabilizing a single-input affine nonlinear system. First, regarding the concept of state feedback linearization, various state transformations are synthesized. In fact, the main benefit of this step is to transform the nonlinear system into an equivalent switched one by using multi-diffeomorphism. In the second part, a Lyapunov analytical bounded control design is developed so as to characterize stability by the use of multiple Lyapunov functions. The design of the proposed approach consists of two main stages. In the first stage, a set of bounded nonlinear state feedback controllers is constructed in a way that it provides an asymptotic stability for each mode. Then, the second stage is aimed to produce a stabilizing switching law that coordinates the transition between the active mode and its appropriate controllers in a way that the process is globally stabilized. Finally, the method is pointed up with an example to demonstrate the applicability of the suggested approach.

Keywords: Switched systems; Multi-diffeomorphism; Bounded nonlinear control; Multiple Lyapunov Functions.

1 Introduction

Traditionally, most of the fundamental control problems have predominantly been concerned with the control of continuous dynamic processes described by ordinary differential equations. Yet, there are many other processes that include discontinuous actuators and physical constraints. Accordingly, the properties of the system instantaneous changes may depend on a complex interaction between the discrete and continuous variables. This, unfortunately, complicates the modelling [5] the analysis and the design [3] of the system. These have extensively been discussed when referring to the hybrid systems because they involve interaction between the continuous and discrete dynamics and the state jumps which altogether lead to the overall system response in time and space. Indeed, amongst the most important classes of such systems are the switched systems [3], [11]. The switched systems are basically the outcome of considering the hybrid systems from the last point of view. More precisely, a switched system is composed of a family of subsystems with a continuous dynamics and a logical law that indicate the active subsystem. In recent years, the research on the switched systems has been the focal point of hundreds of publications, numerous monographs and several comprehensive textbooks, such as [12], [15] which provide an outstanding overview on the switched systems. For this class, the control research is of a growing importance as it helps exploring the conditions of the switched system so as to guarantee the closed loop stability. Actually, the strong desire to implement the control approaches, which allow for an explicit characterization of the stability properties of the switched systems, has motivated the research topic about the design of stabilizing control laws by using the tools of Multiple Lyapunov Functions (MLF) [2], [16]; in other words, one Lyapunov function for each mode. The key feature of the proposed control methodology is the integrated synthesis via MLF. On the one hand, a family of bounded nonlinear feedback controllers enforces stability in the constituent modes and provides an explicit characterization of the feedback stability region for each mode. On the other hand, we attempt to develop switching laws that ensure safe transitions between the modes in a manner that they guarantee stability in the global switched closed-loop system. In whole, this paper is organized as follows. Section 2 is a thorough presentation of the theoretical background of the studied approaches and a review of the basic concepts of input–state feedback linearization. A switching control approach via MLF is described in Section 3. In Section 4, we realize a numerical simulation study in order to validate the proposed approach. Finally, a conclusion is summarized in Section 5.

2 Theoretical Background

2.1 Description of the Studied System

In this work, we have focused on analytical nonlinear continuous systems described by the following state-space equation [9]:

$$\begin{cases} \dot{X} = F(X) + G(X)U \\ y = h(X) \end{cases} \tag{1}$$

with $X \in \Re^n$ is the state vector, n designs the system order, $U \in \Re$ is the input, $y \in \Re^n$ is the system output $F : \Re^n \to \Re^n$, $G : \Re^n \to \Re^n$ are nonlinear vector functions describing the system dynamics. $h : \Re^n \to \Re^n$ is a nonlinear function giving the output expression y. All over this work, the functions F, G and h are assumed to be sufficiently smooth in $\Re^n$ and also differentiable with an unspecified order.

2.2 Inputs-State Feedback Linearization

The design of input-state feedback linearization has been described in many papers [4], [10]. The primary goal, here, is the determination of the relative degree r (the number of times the output has to be differentiated with respect to time before the input appears) which equals to the dimension of the state vector in an operating point x_n. An analytic state feedback transformation $\xi = T(X) = \left[h(X) \quad L_F h(X) \quad \ldots \quad L_F^{r-1} h(X) \right]$ and a static nonlinear state feedback $U = a(X) + b(X)v$ for which the closed loop of system (1), using the feedback in the new coordinates, will be equivalent to a linear model in the following form:

$$\dot{\xi} = \hat{A}\xi + Bv \tag{2}$$

Where $\left(\hat{A}, B\right)$ is a controllable pair of constant matrices of appropriate dimensions. v is a new external input. The following Theorem gives the sufficient condition for the output of the exact linearization:

Theorem 1. [8] *Exact Linearization Problem for the system (1) is solvable near an operating point x_n if and only if the following conditions are satisfied:*

- *The matrix* $\left[G(x_n) \quad ad(F,G)(x_n) \quad \ldots \quad ad^{n-1}(F,G)(x_n) \right]$ *has the rank n.*
- *The distribution* $span\left\{ G \quad ad(F,G) \quad \ldots \quad ad^{n-2}(F,G) \right\}$ *is involutive near* x_n.

If the system (1) satisfies the conditions of Theorem 1, $h(X)$ satisfies this partial derivate equation:

- $L_G L_F^k h(X) = 0, \quad k = 0, \ldots, n-1$
- $L_G L_F^{k-1} h(X) \neq 0$

3 A Switching Control System Approach via Multiple Lyapunov Control Functions

3.1 Formulation of the Proposed Approach

The nonlinear system linearizable by feedback depends on the diffeomorphism and the transient behavior of the loop of regulation will be different. Since the diffeomorphism is not unique, it is shown that a linearizable system feedback has various diffeomorphisms but the best transient behavior is achieved by using the appropriate diffeomorphism.

Using the concept of multi-diffeomorphism in the exact input-state linearization, the dynamics of the nonlinear system (1) has to be transformed into a dynamic switching system which will be transformed as follows:

$$\begin{cases} \dot{x} = f_i\ (x) + g_i\ (x)\ \mathrm{u}_i \\ y = h\ (x) \end{cases} \tag{3}$$

Where $x(t) = [x_1(t),\ldots,x_n(t)] \in \Re^n$ is the state, $u_i = [u_1,\ldots,u_n] \in \Re^n$ is a measurable locally essentially bounded control input taking values in the set $u := \left\{u_i \in \Re^m : \|u_i\| < u_i^{\max}\right\}$ containing the origin. With $\|\ .\ \|$, we note that the Euclidean norm of the signals u, f_i and g_i are a finite family of the smooth vector fields which give rise to the switched nonlinear system (3). $i \in I = \{1,\ldots,N\}$ is a constant function called the switched signal. This is the index set that specifies the active subsystem. The number N of the switches is finite on every bounded time interval. Throughout the paper, we take the notation t_i^k and t_i^{k+1} to denote the t^{th} times that the i^{th} subsystem is switched in and out. We can assume, in the rest of the study, that the continuous state of the i^{th} active mode evolves according to the state equation and the output equation governed for each $t_i^k < t < t_i^{k+1}$. The switching sequence is depicted as shown in Fig.1:

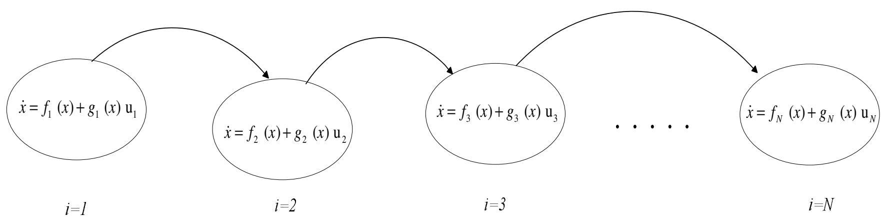

Fig. 1. Concept of the proposed approach.

The key feature of this approach is to propose the switched nonlinear control methodology, for the class of switched nonlinear systems (3), based on the Lyapunov theory which is a useful tool for both the stability analysis and the control theory. One of the most employed stability concepts in the control theory is the MLF. As it will be, later on, used in the control of the switched systems, we will briefly review the main idea of the MLF. In fact, its principle lies in the use of a family of functions named pseudo-Lyapunov functions $\{V_i : i \in I\}$ associated with each field of vectors $\dot{x} = f_i(x)$, to demonstrate stability.

Definition 1. [13] (Pseudo-Lyapunov function)

A pseudo Lyapunov function for the system (3), with $u_i \equiv 0, i \in I$ in an operating point in a stability region of the space ($x_n \in \Omega_i \subset \Re^n$) is a real-valued function $V_i(x)$ defined in a region Ω_i satisfying the following conditions:

– Positive definite: $V_i(x_n) = 0$ and $V_i(x) > 0$ for $x_n \neq x \in \Omega_i$

– derivative defined non-positive: for all x included in the stability region Ω_i

$$\dot{V}(x) = \left(\partial V_i(x)/\partial x\right) f_i(x) \leq 0 \tag{4}$$

We can, then, write the following result:

Theorem 2. [2], [6]

Suppose that $\bigcup \Omega_i = \Re^n$ *and for each vector field* f_i *has an associated Lyapunov-like function* V_i *in the region* Ω_i*, neighborhood* x_n.

For the N-switched nonlinear system (3), with $u_i \equiv 0, i \in I$*, the switching sequence can take the value of* i *only if* $x(t) \in \Omega_i$*, then the value of* V_i *decreases on each interval when the* i^{th} *subsystem is active, more specifically*

$$V_i\left(x\left(t_i^k\right)\right) \leq V_i(x(t_i^{k-1})) \tag{5}$$

We pose t_{ik} *the* k^{th} *switching instant for the sequence. Then, the adjacent of the operating point* x_n *of the system (3), is Lyapunov stable.*

As shown above in theorem 2, the existence of MLF V_i, one for each subsystem f_i, is a necessary and sufficient condition for the stability of every mode i of a system without inputs. In addition, the MLF framework may be used as a tool of control of the switched nonlinear systems with input constraints. The main idea is to use a candidate Lyapunov function for designing the feedback controllers though it is made explicit by introducing the concept of Control Lyapunov Function (CLF) as follows:

Definition 2. [14] (Control Lyapunov Function)

A smooth, proper, and positive-definite function $V:\Re^n \to \Re_+$ called a CLF for a system of the form (1) when there is an admissible value $u^1,\ldots,u^m$ for the controls such that:

$$\inf\left\{L_F V + L_{G_1} V U^1 + \ldots + L_{G_m} V U^m\right\} < 0 \tag{6}$$

Where $L_F V = \partial V/\partial x\, F(x)$, G_k is the k^{th} column of the matrix G.

We can generalize the Definition 2 to a switched nonlinear system as shown in this assumption:

Assumption 1

For every $i \in I = \{1,\ldots,N\}$, *a* Control Lyapunov Function, V_i , exists for system (3).

3.2 Development of the Proposed Approach

In the previous section, we examined how the MLF is used to analyze the stability of the switched nonlinear systems. In this section, we use the MLF as the key for developing a nonlinear control design. The main feature of the proposed approach is not only to synthesize the bounded nonlinear feedback controllers of the individual subsystems, but also to design an appropriate switching scheme that organizes the transition between the different modes and keeps all the system stable. Firstly, we will formulate the control problem and, then, we will propose the switching strategy solutions.

In the present study, we investigate the problem of control, for a switched nonlinear system (3), based on the input–state feedback linearization formalism.

In fact, for the system (3) there is a relative degree $r = n$ in x_n , if it checks the two following conditions, for any x is close to x_n :

- $L_{g_i} L_{f_i}^k \lambda(x) = 0 \qquad i \in I = \{1,\ldots,N\}$ and $0 < k < (n-1)$
- $L_{g_i} L_{f_i}^{(i-1)} \lambda(x) \neq 0$

And a coordinate transformation $\xi = T(x)$ such that the representation of the system of equation (3) in the ξ coordinate takes the form:

$$T_i(x) = \begin{bmatrix} T_i^1(x) \\ T_i^2(x) \\ \vdots \\ T_i^n(x) \end{bmatrix} = \begin{bmatrix} \xi_i^1(x) \\ \vdots \\ \xi_i^{r-1}(x) \\ \xi_i^n(x) \end{bmatrix} = \begin{bmatrix} \lambda(x) \\ L_{f_i}\lambda(x) \\ \vdots \\ L_{f_i}^{n-1}\lambda(x) \end{bmatrix} \tag{7}$$

The resulting system with the transformed variables (7) can then be written as:

$$\begin{cases} \dot{\xi}_i^1 = \xi_2 \\ \vdots \\ \dot{\xi}_i^{r+1} = \xi_\gamma \\ \dot{\xi}_i^r = L_{f_i}^n \lambda(x) + L_{g_i} L_{f_i}^{n-1} \lambda(x) u_i \\ y = \xi_1 \end{cases} \tag{8}$$

where $L_{f_i}\lambda(x) = (\partial\lambda/\partial x) f_i(x)$ is the Lie derivative of $f_i(.)$ with $L_{g_i} L_{f_i}^{n-1}\lambda(x) \neq 0$ for all $x \in \Re^n$, $x = T_i^{-1}(\xi_i)$ and u_i is the new input control.

The first step of the control problems is to synthesize, via MLFs V_i existing for each $i \in I = \{1,\ldots,\mathrm{N}\}$, a family of N-bounded nonlinear continuous feedback controllers that enforces the asymptotic stability for their respective closed-loop subsystems. To achieve this end, this control law can be constructed:

$$u_i = \begin{cases} -k(x, u_i^{\max})(\left(L_{g_i} V_i\right)^T(x)), & \left\|\left(L_{g_i} V_i\right)^T(x)\right\| \neq 0 \\ 0 & \left\|\left(L_{g_i} V_i\right)^T(x)\right\| = 0 \end{cases} \tag{9}$$

where

$$k(V_i, u_i^{\max}) = \begin{cases} \dfrac{\theta_i(x) + \sqrt{\alpha_i^2(x) + \left(u_i^{\max}\left\|\left(L_{g_i} V_i\right)^T(x)\right\|\right)^4}}{\left\|\left(L_{g_i} V_i\right)^T(x)\right\|^2 \left(1 + \sqrt{1 + \left(u_i^{\max}\left\|\left(L_{g_i} V_i\right)^T(x)\right\|\right)^2}\right)} & \left\|\left(L_{g_i} V_i\right)^T(x)\right\| \neq 0 \\ 0 & \left\|\left(L_{g_i} V_i\right)^T(x)\right\| = 0 \end{cases}$$

The vector of the manipulated inputs u_i is bounded by $\|u_i\| \leq u_i^{\max}$ where the note $\|\ .\ \|$ is the Euclidean norm of the input and $u_i^{\max}$ is a real positive number. $L_{g_i} V_i = \left[L_{g_i^1} V_i \ldots L_{g_i^m} V_i\right]$ is a row vector, where $L_{g_i} V_i$ is the Lie derivatives of the control Lyapunov function V_i for the i^{th} mode along the column vectors of the matrix g_i, $\theta_i(x) = L_{f_i} V_i + \rho V_i$ and $\rho > 0$.

So, the behavior is globally input-output linearized according to the previous definition as it is allowed to synthesize ξ controllers ensuring the stability of the closed loop system. For this reason, we adopt the following notation $e_i = \xi_i - v^{(k-1)}$ where $e = [e_1 e_1 \ldots e_r]^T$, $v = \left[v v^{(1)} \ldots v^{(r-1)}\right]^T$ is a generalized reference input where $v^{(k)}$ k$^{\text{th}}$ is the time derivative of this latter. Consequently, one may prove that the ξ-subsystem (8) will be equivalent to the form:

$$\begin{cases} \dot{e} = \psi_i(e,v) + \omega_i(e,v)u_i, \qquad \text{i=1,\ldots,N} \\ \psi_i(e,v) = Ae + BL_{f_i}^r h(x) \\ \omega_i(e,v) = BL_{g_i}^r h(x) \end{cases} \tag{10}$$

Where ψ_i and ω_i are $r \times 1$ are the vector fields, A and B are respectively $r \times r$ matrix and $r \times 1$ vector with the form:

$$A = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & & & & \vdots \\ 0 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 0 \end{bmatrix} \text{ and } \mathrm{B} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \\ 1 \end{bmatrix} \tag{11}$$

The previous compact form allows constructing a CLF for each mode of the switched system (10) by the use of a quadratic Lyapunov function $\overline{V_i} = e^T P_i e$ where P_i is a positive definite matrix chosen to satisfy the following Ricatti inequality:

$$A^T P_i + P_i A - P_i B B^T P_i < 0 \tag{12}$$

Referring to the second control objective, we must integrate the synthesis, bounded nonlinear feedback controllers, given in equation (9), and the switching laws that organize the transitions between the constituent modes and their respective controllers. On the whole, we will use multiple CLFs to design a family of bounded controllers, with its estimated regions of stability, in a way that it allows us to present the switching rule to any of the bounded controllers at a given time. Theorem 3 below summarizes the proposed switching control strategy. The main idea provided in this theorem is the formulation of the family of bounded feedback continuous controllers together with the appropriate switching rules to govern the transitions between the various closed-loop modes in a way that they guarantee the desired properties in the constrained switched closed-loop system.

Theorem 3. [7]

Consider the switched nonlinear process (3), for all $i \in I$ *, under the family* u_i *of the bounded nonlinear feedback controllers given by the equation (9), where* V_i *is a control Lyapunov function for the* i^{th} *subsystem. We notice that the closed-loop state trajectory, x, evolves within the state-space region described by this set:*

$$D = \left\{ x \in \Re^n : L_{f_i} V_i(x) + \rho_i V_i(x) \le u_i^{\max} \left\| \left(L_{G_i} V_i(x) \right)^T \right\| \right\}$$

We construct the largest invariant subset $\Omega_i^*\left(u_i^{\max}\right)$ *using the level sets of* V_i *:*

$$\Omega_i^*\left(u_i^{\max}\right) = \left\{ x \in \Re^n : V_i(x) \le \gamma_{x,i} \right\}$$

Where $\gamma_{x,i}$ *is the largest number for which* $D_i \supset \Omega_i^*\left(u_i^{\max}\right)$. *Without loss of generality, we assume that* $x(x_n) \in \Omega_i^*\left(u_i^{\max}\right)$. *If, at any given time* Γ, *the following conditions hold:*

$$x(\Gamma) \in \Omega_i^*\left(u_i^{\max}\right) \tag{13}$$

$$V_j\left(x(\Gamma)\right) < V_j\left(x(t_{j^*})\right) \tag{14}$$

For some $j \in I, j \neq i$ *,where* $t_{j^*} < \Gamma$ *is the time when the* j^{th} *subsystem was last switched in, then setting* j *, for* $t \ge \Gamma^+$ *, guarantees that the origin of the switched closed-loop system is asymptotically stable.*

4 Illustrative Example and Simulation Studies

In this section, the simulation study is presented to demonstrate the implementation, to evaluate the effectiveness of the proposed switching control strategy based on the concept of multi-diffeomorphism as well as Multiple CLF and to test its stability. We consider the example of a nonlinear system described by the following equation:

$$\begin{cases} \dot{X}_1 = X_1^2 + X_2 \\ \dot{X}_2 = X_1 + X_2 + U \end{cases} \tag{15}$$

The system (15) can be written as the form of equation (1). Where $F(X) = \begin{bmatrix} X_1^2 + X_2 & \mathrm{X}_1 + X_2 \end{bmatrix}^T$, $G(X) = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$

The study is considered in the neighborhood of the instable operating point $x_n = (1, -1)$.

An exact input-state linearization is used to find a non-linear transformation such that in the new coordinate system the model can be completely linear using the feedback. The dynamics of the system (15) can be improved by using the multi-diffeomorpism in linearization. According to the change of coordinate equation (7) the process will be transformed under various forms that can be stated as model (3), where $i = 1,2$.

The first diffeomorphism is spelled as:

$$T_1(x) = \begin{bmatrix} x_1 \\ x_1^2 + x_1 \end{bmatrix} \tag{16}$$

Then, from the conditions of theorem 1 we can obtain another diffeomorphism:

$$T_2(x) = \begin{bmatrix} 1 + x_1{}^2 \\ \left(x_1^2 + x_2\right) 2x_1 \end{bmatrix} \tag{17}$$

By the use of the equation (9), we designed the bounded controller to accomplish our control objectives which are:

- to stabilize the parameters in the operating point beginning by mode 1
- to maintain this state when the system switches to mode 2

The system (15), describing the dynamics, can be obtained for controller design:

We construct two bounded controllers for each mode of the form of equation (18) using two quadratic Lyapunov function computed using the linearized system. We choose $\overline{V_1} = \overline{V_2} = 1/2\left(x_1 + x_2\right)^2$

$$u_i = \frac{L_{f_i}\overline{V_i} + \rho\overline{V_i} + \sqrt{(L_{f_i}\overline{V_i} + \rho\overline{V_i}) + \left(u_i^{\max}\left\|\left(L_{g_i}\overline{V_i}\right)\right\|\right)^4}}{\left\|\left(L_{g_i}\overline{V_i}\right)\right\|^2\left(1 + \sqrt{1 + \left(u_i^{\max}\left\|\left(\left(L_{g_i}\overline{V_i}\right)\right)\right\|\right)^2}\right)}\left(L_{g_i}\overline{V_i}\right) \tag{18}$$

Where $L_{f_i}\overline{V_i} = \left[\partial\overline{V_i}/\partial e\right] f_i$ and $L_{g_i}\overline{V_i} = \left[\partial\overline{V_i}/\partial e\right] g_i$, i=1,2

The CLF cannot stabilize the full system according to theorem 2 we should use for:

- mode1:
  $V_1 = 1/2\,\sigma_1 (x_2 + 1)^2 + 1/2\,\sigma_2 (x_1 - 1)^2, \quad \sigma_1 = 0.94$ and $\sigma_2 = 0.23$
- mode 2:

$$V_2 = 1/2\,\sigma_3 (x_2 + 1)^2 + 1/2\,\sigma_4 (x_1 - 1)^2, \quad \sigma_3 = 0.95 \text{ and } \sigma_4 = 1.76$$

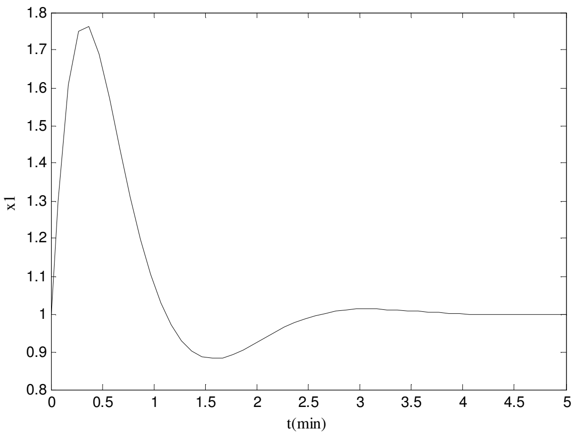

Fig. 2. Evolution of the closed-loop state x_1 when the system is initialized in mode 1(without switch)

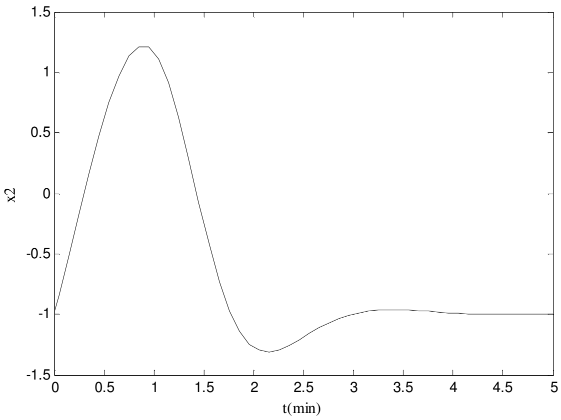

Fig. 3. Evolution of the closed-loop state x_2 when the system is initialized in mode 1(without switch)

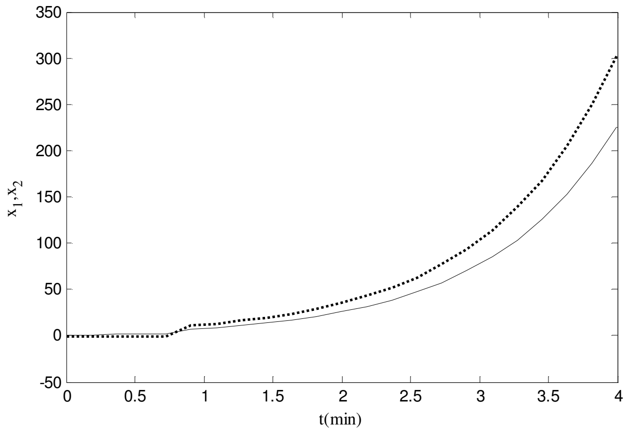

Fig. 4. Closed-loop state
when the system switches to mode 2 at *t* =1 min: x_1(solid) and x_2 (dotted).

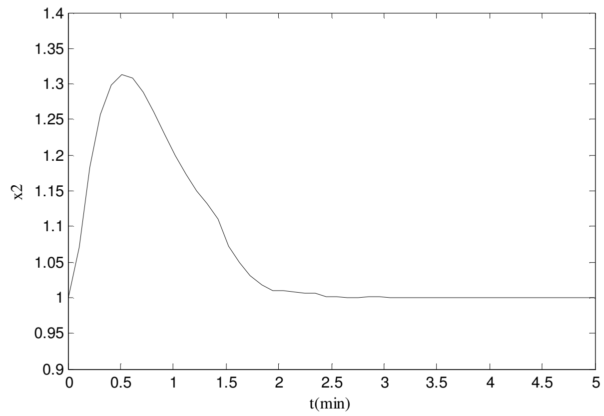

Fig. 5. Closed-loop state
when the system switches to mode 2 at *t* =0.5 min.

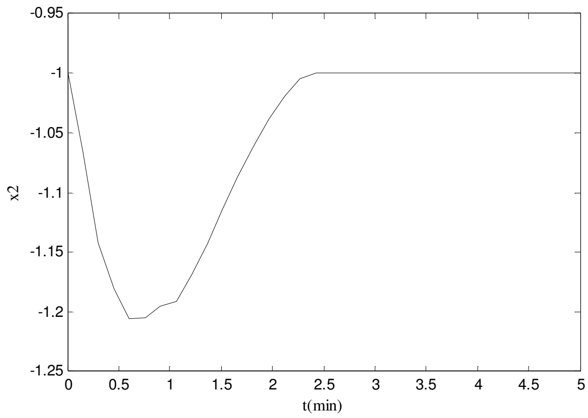

Fig. 6. Closed-loop state
when the system switches to mode 2 at t =0.5 min.

Here, we present the simulation of the control objective which stabilizes the states of the system in the unstable operating point and illustrates the behavior of the controlled system. Fig. 2 and Fig.3 show the convergence of the states to the desired values when the system is operated in mode 1 for all times (with no switching).

According to these figures, it is clear that the control provided for this mode could successfully stabilize the system in the desired unstable operating point.

But when the system switches to the second mode at an arbitrarily given moment $t=1$ *min*, the parameters of the system diverge and the control fails to stabilize the states according to Fig. 4. This phenomenon can be explained by the fact that the parameters are outside the stability region of mode 2. In order to successfully put the system back in its stable condition, we use the switching diagram shown in theorem 3.

This objective is illustrated in Fig. 5 and Fig. 6 where the system is initialized to the first mode. At an arbitrary moment $t=0.5$ *min* existing in the stability region of the second mode (according to switching condition of the theorem), the control switches to mode 2. But this time the goal of the control is achieved: the controllers successfully handle the system and, also, remain stable in the neighborhood of the point which is desired throughout the switching phase.

6 Conclusion

In This work, we considered the problem of stabilization of nonlinear systems. The inner loop was designed based on the new concept of multi-diffeomorphism. By using state feedback linearization an equivalent switched linear model for the system is constructed. The outer loop is a switched linear controller which guarantees a global stability. To sum up, we use Lyapunov function $\overline{V_i}$ in designing the bounded controllers for e-subsystems that stabilize the full e-interconnection for each mode. We also employ the Lyapunov function V_i to implementing the switching rules. These Lyapunov functions are used in verifying the switching conditions at any given time. What is more, we demonstrated the application of the propose controller designs through a numerical example.

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